



Particle Kinematics in Spherical Coordinates

$$\begin{aligned}\hat{r} &= \cos \theta \cos \phi \hat{i} + \sin \theta \cos \phi \hat{j} + \sin \phi \hat{k} \\ \hat{\theta} &= -\sin \theta \hat{i} + \cos \theta \hat{j} \\ \hat{\phi} &= -\cos \theta \sin \phi \hat{i} - \sin \theta \sin \phi \hat{j} + \cos \phi \hat{k}\end{aligned}$$

Angular velocity of the coordinates system:

$$\bar{\omega} = \dot{\theta} \sin \phi \hat{r} - \dot{\phi} \hat{\theta} + \dot{\theta} \cos \phi \hat{\phi} \quad (1)$$

Position:

$$\bar{r} = r \hat{r} \quad (2)$$

Velocity:

$$\bar{v} = \dot{\bar{r}} = \frac{\partial \bar{r}}{\partial t} + \bar{\omega} \times \bar{r} = \dot{r} \hat{r} + r \dot{\theta} \cos \phi \hat{\theta} + r \dot{\phi} \hat{\phi} \quad (3)$$

Acceleration:

$$\begin{aligned}\bar{a} = \dot{\bar{v}} = \ddot{\bar{r}} &= \left(\frac{\partial}{\partial t} + \bar{\omega} \times \right) \left(\frac{\partial \bar{r}}{\partial t} + \bar{\omega} \times \bar{r} \right) = (\ddot{r} - r \dot{\theta}^2 \cos^2 \phi - r \dot{\phi}^2) \hat{r} \\ &+ (2\dot{r} \dot{\theta} \cos \phi - 2r \dot{\theta} \dot{\phi} \sin \phi + r \ddot{\theta} \cos \phi) \hat{\theta} + (2\dot{r} \dot{\phi} + r \dot{\phi}^2 \sin \phi \cos \phi + r \ddot{\phi}) \hat{\phi} \quad (4)\end{aligned}$$